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Supersonic Flow Development in Slotted Wind Tunnels

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The development of test section slot shapes for achieving smooth supersonic Mach number distribution without overexpansion or waviness has, in the past, been largely an experimentally iterative or "cut-and-try" procedure for each wind tunnel. To overcome the obvious disadvantages of time and expense involved in such an experimental approach, a simple analytical method to predict the supersonic flow development in a two-dimensional slotted transonic wind tunnel has been developed and validated. The well-known method of characteristics is used with the constraint that it be compatible with the quadratic cross-flow pressure drop boundary condition at the slotted wall. While doing that, an insight has been gained into the flow mechanism which causes overexpansion with some slot shapes. As a consequence of the success of the analysis method, a design method has been developed on similar lines, to obtain slot shapes for prescribed smooth supersonic flow development at a design Mach number.

Nomenclature

a = right running characteristics
b = width of the tunnel throat; also left running characteristic
C_{p_w} = local wall pressure coefficient with plenum static pressure and local dynamic pressure as reference values,

$$\frac{p_w - p_p}{\frac{1}{2}\rho_w U_w^2} = \frac{2}{\gamma M_w^2}\left\{1 - \frac{p_p}{p_t}\left(1 + \frac{\gamma - 1}{2} M_w^2\right)^{(\gamma/\gamma - 1)}\right\}$$

h = height of the tunnel throat
K = slot "orifice" coefficient. Slot width reduction factor taken constant for the entire slot length
L = length prescribed for $(\nu+\theta)_w$ to reach ν_d
M = Mach number
p = static pressure
p_p = plenum static pressure
p_t = stagnation pressure of the tunnel freestream
R = open area ratio of the top and bottom slotted walls (based on the width b of the top and bottom walls only and not the total perimeter)
U_w = local velocity at the wall
x = distance measured downstream from the slot origin
X = x/h
y = vertical distance from the centerline of the tunnel
γ = ratio of specific heats
θ = local inclination of the flow with respect to the tunnel centerline
θ_w = inclination of the flow at the homogeneous wall with respect to the centerline (outflow from test section to plenum is considered positive)
θ_s = flow inclination at the center of the slot with respect to the effective wall slope (outflow from test section to plenum is considered positive)
μ = Mach angle of the local flow
ν = Prandtl-Meyer angle of the local flow
ρ = density

Subscripts

cl = centerline
d = design value
p = plenum
sw = sidewall
t = stagnation conditions
tw = top wall
w = equivalent homogeneous wall

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Introduction

SLOT shapes for transonic wind tunnels are designed in many instances from the consideration of generating smooth supersonic flow in the test section. In the past, such a design has been carried out through extensive experimental investigations with several slot shapes, on a "cut-and-try" basis, involving considerable expense and time (see Ref. 1, for example). The present study was, therefore, taken up with the objective of reducing this expense and time by developing the slot shapes essentially through analytical means and limiting the experiments for refinement only.

This paper deals with the following: 1) development and validation of a simple analysis method for predicting the supersonic flow development in a two-dimensional slotted transonic wind tunnel for a given slot shape; 2) understanding of the flow mechanism which causes overexpansion with certain slot shapes; and 3) development of a design method on lines similar to the analysis method for obtaining desired open-area ratio variation for a prescribed smooth supersonic flow development at a design Mach number and off-design performance of such slot shapes.

Development of the Characteristic Network

The approach used in both the analysis method and the design method is based essentially on the well-known method of characteristics. The commonality of the approach for both the methods is indicated in this section.

Figure 1 shows the development of the characteristic network. The slotted wall with discrete slots is replaced by an equivalent homogeneous wall. To develop the characteristic network, it is necessary that the boundary condition which provides a relation between ν_w, the Prandtl-Meyer angle corresponding to the Mach number M_w at the homogeneous wall, and θ_w, the flow inclination at the homogeneous wall with respect to the centerline, be known. In the analysis method this is obtained from the cross-flow pressure drop boundary condition at the wall and in the design method this is obtained by prescribing a smooth $(\nu+\theta)_w$ distribution. For

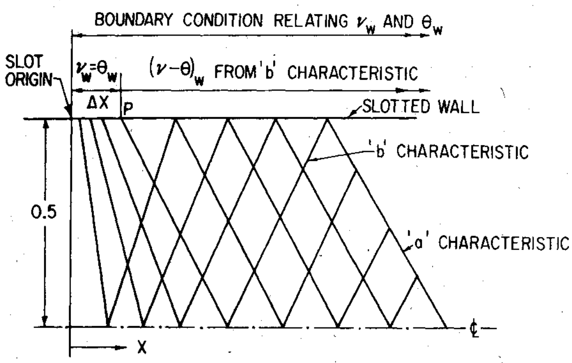

Fig. 1 Characteristic network.

the development of the characteristic network, it is sufficient if a boundary condition at the wall relating ν_w and θ_w is known.

At the outset it may be mentioned that the development of the characteristics network is analogous to that in a solid two-dimensional supersonic nozzle except for the fact that in the solid nozzle, the boundary condition at the wall amounts to θ_w being given, whereas in the present case, the boundary condition is in terms of a relation between ν_w and θ_w. To start the characteristic network, a normal sonic line at the slot origin is assumed and the flow at the wall is assumed to undergo a Prandtl-Meyer expansion over an initial small length ΔX of the order of $0.02h$, where h is the test section height. This, as already stated, is analogous to the method used in a solid supersonic nozzle. This means $\nu_w=\theta_w$ in this region. This equality between ν_w and θ_w in this region, in conjunction with the known boundary condition relating ν_w and θ_w, provides the solution for the individual values of ν_w and θ_w in this wall region. Knowing this, it is straightforward to develop the characteristic network between the normal sonic line and the a characteristic from point p at ΔX (see Fig. 1).

The solution for ν_w and θ_w beyond ΔX is obtained from the requirement that they simultaneously satisfy the requirement of known wall boundary condition as well as the requirement that $(\nu-\theta)_w$ correspond to the known value of $(\nu-\theta)$ on the b characteristic meeting the wall. On this basis, if one a characteristic from the wall to the centerline is known, then the subsequent a characteristics can be easily obtained. The a characteristic from point p being known, the downstream network can be computed easily as long as the boundary condition at the wall is known. Reference 2 gives a more detailed description of the application of the method of characteristics to the analysis problem. An important point to note, which will be referred to often, is that along the a characteristic $(\nu+\theta)$ at wall, manifests as ν on the centerline, since θ is zero on the centerline.

Analysis Method

The analysis method deals with the prediction of supersonic flow development in a two-dimensional slotted transonic tunnel given the slot shape or the open-area ratio variation and M_p, the plenum Mach number corresponding to plenum static pressure. As indicated in the previous section on the development of the characteristic network, if one can arrive at a homogeneous boundary condition at the wall which would give a relation between ν_w and θ_w, then the whole flowfield can be determined easily. We shall now discuss such a boundary condition.

Boundary Conditions at the Slotted Wall

The boundary conditions at the slotted wall have been a subject for study for many years by many workers,[3-8] but is not yet fully resolved. All the theoretical studies carried out are based on inviscid subsonic linear theory with the added assumption of small perturbation. The purpose of most of these studies has been to look into the interference problems associated with model testing. Therefore, the assumption of small perturbation at the wall is reasonably valid. In the present study, the flow is supersonic and, hence, the validity of mathematical models based on subsonic linear theory is questionable. Further, considerable amount of outflow from the test section into the plenum is involved in generating supersonic Mach numbers of the order 1.2-1.3 and, hence, the assumption of small perturbation is also questionable. The experimental results of Gardnier and Chew quoted in Ref. 8 indicate that the cross-flow pressure drop characteristics are linear for small cross-flow velocities with an additional quadratic term whose contribution becomes significant only at large cross-flow velocities. However, Berndt[5] from his inviscid theoretical analysis concludes that only quadratic terms (apart from the terms which appear due to flow curvature) can be present and validates his hypothesis from his experimental results. However, his experimental setup was such that there was considerable outflow into the plenum even under tunnel empty conditions (without model) and, hence, it is difficult to draw conclusions for small cross flow. Thus, it appears that this issue is not yet resolved. Jacocks[9] has shown that the boundary-layer growth on the perforated wall significantly affects its cross-flow characteristics. Similar detailed studies for the case of slotted walls have not been carried out but it is reasonable to expect that even in this case the boundary-layer growth on the walls would affect the cross-flow characteristics. In this regard, a new hypothesis on the cross-flow characteristics of slotted walls has been proposed by the first author in Ref. 10 where it is indicated that even for wide slots, where one normally expects the cross-flow phenomena to be inviscid, viscous effects are present for small cross-flow velocities because of the presence of the shear layer in the slot region. Because of this shear layer it is hypothesized that the cross-flow characteristics (in the absence of curvature) could be linear for small cross-flow velocities and quadratic for large cross-flow velocities. Because of so many uncertainties it was considered best to take a simple rational engineering approach in treating the boundary condition as indicated subsequently.

It is assumed that the flow, at least at the center of the slot, is inviscid implying that the stagnation pressure of the flow at the center of the slot entering the plenum chamber is equal to the stagnation pressure of the tunnel flow. Therefore,

$$p_w\left(1+\frac{\gamma-1}{2}M_w^2\right)^{(\gamma/\gamma-1)}=p_p\left(1+\frac{\gamma-1}{2}M_p^2\right)^{(\gamma/\gamma-1)}=p_t \quad (1)$$

where p_w is the wall static pressure, γ the ratio of specific heats, M_w the Mach number at the wall, p_p the plenum static pressure, and p_t the tunnel stagnation pressure. It is further assumed that the local longitudinal velocity along the wall remains the same across the width of wall including the slot region and the effect of the difference between wall pressure and the plenum pressure is to introduce, in the jet flow entering the plenum, a component of the velocity normal to wall (see Ref. 10). These assumptions lead to the relation

$$C_{p_w}=\frac{p_w-p_p}{\frac{1}{2}\rho_w U_w^2}=\frac{2}{\gamma M_w^2}\left\{1-\left(1-\frac{\gamma-1}{2}M_w^2\tan^2\theta_s\right)^{(\gamma/\gamma-1)}\right\} \quad (2)$$

where C_{p_w}, ρ_w, U_w, and θ_s are, respectively, the wall pressure coefficient, local density at the wall, local velocity at the wall, and the flow inclination at the center of the slot. It may be noted that for the incompressible case, corresponding to M_w tending to zero, Eq. (2) reduces, in the limit, to

$$C_{p_w}=\tan^2\theta_s \quad (3)$$

a result which is to be expected.

It is interesting to note that even for $\tan\theta_s$ of the order of 0.4 and M_w of the order of unity, Eq. (3) is a good approximation to Eq. (2), the difference in the value of C_{p_w} being within 4%. Therefore, as a matter of convenience, Eq. (3) was used for obtaining the subsequent homogeneous wall boundary condition.

It may be pointed out that C_{p_w} is determined from the expression

$$C_{p_w} = \frac{2}{\gamma M_w^2}\left\{1 - \frac{p_p}{p_t}\left(1+\frac{\gamma-1}{2}M_w^2\right)^{(\gamma/\gamma-1)}\right\} \tag{4}$$

Since p_p/p_t is a given quantity, C_{p_w} is a function of M_w or the corresponding Prandtl-Meyer angle ν_w only.

If the wall with a discrete number of slots is replaced by a homogeneous wall, then, using Eq. (3), θ_w, the inclination of the flow through the homogeneous wall with respect to the wall, is given by

$$\theta_w \simeq \tan\theta_w = KR\tan\theta_s = KR\sqrt{C_{p_w}} \tag{5}$$

where R is the local geometric open-area ratio of the slotted wall and K is the slot orifice coefficient used to take account of the vena contracta of the cross-flow jet, as well as the viscous effects introduced by the boundary layer on the slats entering the slots. Though not essential, for the present, we have considered only those cases where proper geometric wall divergence has been provided to compensate for the boundary-layer growth and the effective slope of the homogeneous wall can be taken to be zero. Then θ_w will be the inclination of the flow with respect to the centerline. Note that θ_w is a function of ν_w only since R and p_p/p_t are known and K is assumed. Thus, the cross-flow pressure drop boundary condition of the slotted wall gives a relation between ν_w and θ_w and, consequently, allows the characteristic network to be developed. Thus the flowfield throughout the test section can be determined.

The authors realize that in the preceding boundary condition, the influence of the streamline curvature and varying width of the slot has been neglected. It may be mentioned that, in fact, some trials were made taking into account the curvature term also, using the expressions given in Ref. 5. The difference in the result was marginal. Therefore, it was felt that when an empirical factor such as K was being introduced into the analysis, refinements of including the curvature term were not called for. Since the proper boundary condition for a slotted wall, in the opinion of the authors, is still an unresolved problem, the authors felt that for an engineering approach, the simplest possible boundary condition would suffice.

Validation of the Analysis Method

To check the validity of the method, the flow development in the Langley 8-ft transonic pressure tunnel (8-ft TPT) and the Langley diffuser flow apparatus (DFA) were calculated.

8-ft Transonic Pressure Tunnel

This tunnel has a square cross section of 2.172×2.172 m with solid sidewalls, and slotted top and bottom walls with 4 slots each. In the initial stages of the development of this tunnel, a number of slot shapes were tested to arrive at the optimum. For this study, we have considered the two widely different slot shapes 2 and 2f (shown in Fig. 2a). Slot 2 (for which the flow development was unsatisfactory) was one of the earliest tried. Slot 2f was the final satisfactory choice with which the facility presently is running. The primary purpose of the analysis reported herein was to determine whether the analysis method would predict the experimental results obtained during the calibration of the two widely different slot shapes. The walls of the 8-ft TPT are diverged to compensate for the boundary-layer growth.

Diffuser Flow Apparatus

The DFA is a transonic facility having a square test section whose dimensions are 46.37×46.37 cm. Experimental calibration data obtained in this facility (with sidewalls solid and slotted top and bottom walls with 6 slots each) are considered here for comparison with the anlysis method. The slot geometry is shown in Fig. 2b. In these tests, the sidewalls were kept parallel and the top and bottom slotted walls were diverged to compensate for the boundary-layer displacement effect.

Results and Discussions

The computed results presented here have been obtained assuming that the effective slope of the slotted walls was zero since the walls of both the 8-ft TPT and DFA are diverged to compensate for the boundary-layer growth.

Flow Development in the 8-ft TPT

Typical comparisons of the theoretical and experimental results (unpublished) obtained for slot shape 2 and slot shape 2f at different plenum Mach numbers are presented in Figs. 3 and 4. Figure 3 refers to experimental results obtained without plenum suction and Fig. 4 refers to experimental results obtained with plenum suction. From these figures, it can be inferred that a value of slot orifice coefficient K of the order of 0.8-0.9 gives good agreement between theory and experiment. The quantitative differences in the results, by taking the value of K to be 0.8 or 0.9, are within acceptable limits. With either of these values of K, the theory predicts extremely well all the qualitative trends shown in the experimental data.

The hump in the centerline Mach number distribution obtained with slot shape 2 at $M_p = 1.218$ is clearly seen in Fig. 3a. The fact that the maximum centerline Mach number for slot shape 2 is much higher than the plenum Mach number is predicted by theory. Similar trends obtained experimentally for slot shape 2 at other plenum Mach numbers (not presented here) were also predicted equally well by theory.

The theoretical results obtained for slot shape 2f predicted satisfactorily both the centerline Mach number distribution and the top wall Mach number distribution for several values of plenum Mach number. Typical comparisons for $M_p = 1.222$ are shown in Figs. 3b and 3c. The relatively uniform centerline Mach number distribution obtained with slot shape 2f without plenum suction is predicted by theory.

Figure 4 shows a comparison of the theoretical results with experimental data obtained with slot shape 2f with plenum suction. It is seen that the analysis method is capable of following the unusually distorted Mach number distributions

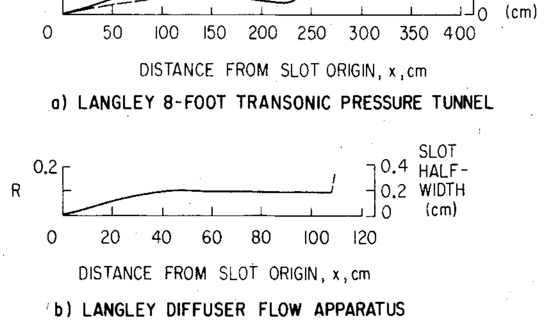

Fig. 2 Slot geometries used in the analysis.

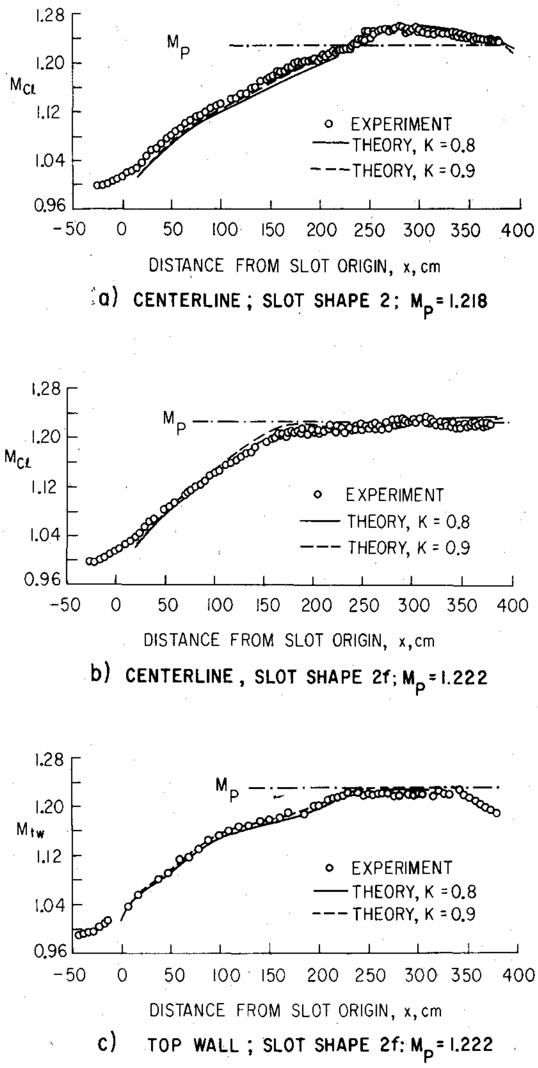

Fig. 3 Mach number distributions in the Langley 8-ft transonic pressure tunnel without plenum suction.

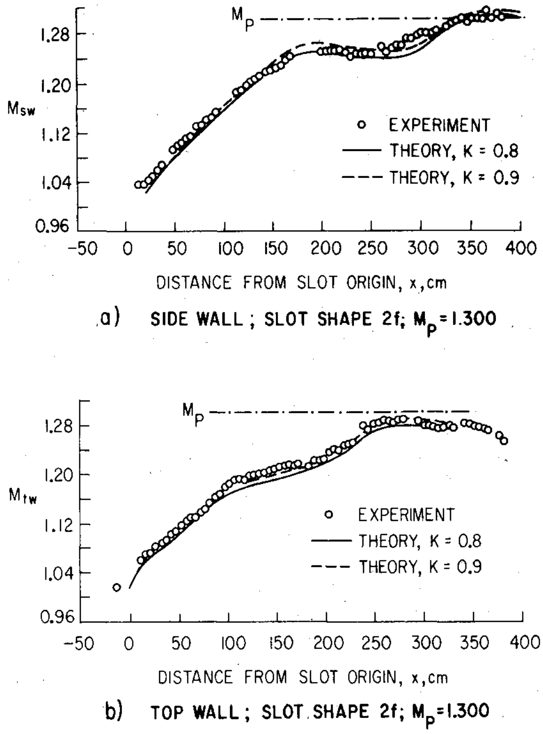

Fig. 4 Mach number distribution in the Langley 8-ft transonic pressure tunnel with plenum suction.

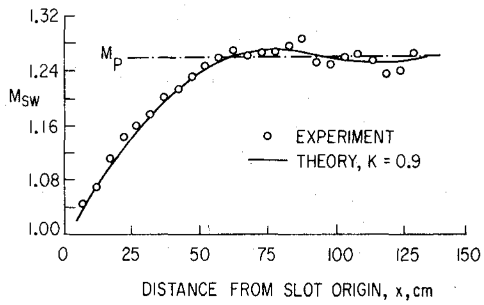

Fig. 5 Sidewall Mach number distribution in the Langley diffuser flow apparatus for $M_p = 1.257$.

obtained on the centerline of the sidewall as well as on the top wall. The plateau in the Mach number distributions can be attributed to the gradual constriction in the slot shape downstream of approximately the 100-cm station. The constriction, which leads to a minimum open area ratio between the 200- and 230-cm stations for slot shape 2f (Fig. 2a), limits the outflow through the slots and, thus, limits the Mach number to which the test section flow can expand. The subsequent increase in the slot opening beyond the 230-cm station allows greater outflow through the slots and a consequent increase in the test section Mach number. The fact that the theory does predict these unusual trends remarkably well does indicate that the essential physics of the flow development process in a slotted tunnel is contained in the analysis method.

Flow Development in DFA

A typical comparison between theory and experiment for the centerline Mach number distribution obtained in DFA at $M_p = 1.257$ is presented in Fig. 5. Similar agreement between theory and experiment was obtained at other plenum Mach numbers, but the results are not presented here. Although there is considerable scatter and undulations in the experimental data, it appears reasonable to infer that the qualitative and quantitative agreement between theory and experiment is satisfactory and that the results further reinforce the validity of the method.

Flow Mechanism Causing Overexpansion

As the Mach number along the slotted wall increases from 1.0 at the throat to the value corresponding to the plenum Mach number M_p, the outflow through the slots should gradually go to zero. On this basis, one expects the Mach number on the slotted wall not to go above M_p. To check the hypothesis, the theoretical Mach number distribution on the top slotted wall obtained for the 8-ft TPT with slot shape 2 (Fig. 6) was considered. This particular example was chosen because the Mach number on the centerline exceeds the value of M_p by the largest margin of all the cases considered herein and is, therefore, the severest test case for the hypothesis. From Fig. 6, it is clear that the hypothesis on the top wall Mach number distribution is indeed valid. The centerline Mach numbr distribution predicted by theory (which was shown in Fig. 3a to be in good agreement with experimental data) is also shown in Fig. 6. It is obvious that, even though the wall Mach number does not exceed M_p, the centerline Mach number can exceed it by a large margin. By careful study, the reason for this becomes quite apparent.

One may recall that the flow was developed by finding the solution at successive mesh points along successive a characteristic lines (see Fig. 1). Along these a lines, the quantity $(\nu+\theta)_w$ is equal to ν_{cl} (since $\theta=0$ on the centerline).

Therefore, even if M_w on the wall does not exceed M_p, that is, ν_w does not exceed ν_p (corresponding to M_p), it is possible for $(\nu+\theta)_w$ to exceed ν_p. If that happens, ν_{cl} will exceed ν_p; that is, M_{cl} will exceed M_p. For a particular value of ν_w (less than ν_p), the higher the open-area ratio, the higher will be the value of the flow angle θ_w [see Eq. (5)]. Therefore, one can readily see that if overexpansion has to be reduced, the open-area ratio has to be reduced.

To gain further insight, a slotted wall having constant open-area ratio along its entire length was considered. The manner in which $(\nu+\theta)_w$ would vary as ν_w increased from 0 to ν_p was studied. For simplicity, K was assumed equal to 1.0. From Eq. (5), for the assumed constant open-area ratio, variation of θ_w with ν_w as ν_w increases from 0 to ν_p can be found. (C_{p_w} is purely a function of ν_w and ν_p.) Figure 7 shows the variation of $(\nu+\theta)_w$ with ν_w for open-area ratios of 0.05 and 0.10. In the figure, the Mach number scale corresponding to ν is also indicated. It is clearly seen that as ν_w approaches ν_p, $(\nu+\theta)_w$ exceeds ν_p. The extent to which this occurs is indicated by the amount that the solid and dashed curves exceed the $M=1.20$ ordinate. For $R=0.05$, the maximum $(\nu+\theta)_w$, which results in ν_{cl}, corresponds to a Mach number of about 1.205 for $M_p=1.20$. Similarly, for $R=0.10$, the maximum $(\nu+\theta)_w$ corresponds to a Mach number of about 1.22, which is 0.02 above $M_p=1.20$.

It may be noted, from Figs. 2a and 6 that the open area ratio of slot shape 2, where ν_w is close to ν_p, is of the order of 0.2. Consequently, the maximum M_{cl} (which is about 1.250 exceeds $M_p=1.218$ by 0.032, for $K=0.8$. It is clear that if one desires to avoid the overexpansion, one must provide a progressively smaller open-area ratio as the Mach number on the slotted wall approaches the plenum Mach number. From the shape of slot 2f (Fig. 2a), which was found to be a satisfactory shape for $M_p=1.20$, it is clear that its success is due to the reduction in the open-area ratio to a low value of 0.04 as ν_w approaches ν_p (see Figs. 2a and 3c). Once ν_w has reached ν_p, opening up the slots further downstream for other considerations (such as wind tunnel wall interference on test models) does not significantly alter the downstream flow development. This is true because there would be little or no pressure difference across the slots to cause any significant flow through them.

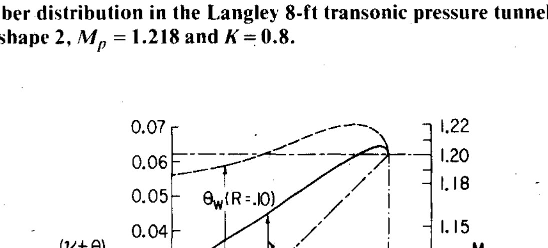

Fig. 6 Comparison of the theoretical centerline and top wall Mach number distribution in the Langley 8-ft transonic pressure tunnel with slot shape 2, $M_p=1.218$ and $K=0.8$.

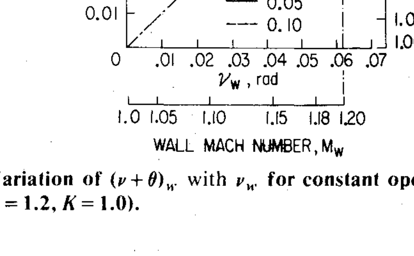

Fig. 7 Variation of $(\nu+\theta)_w$ with ν_w for constant open-area slotted walls ($M_p=1.2$, $K=1.0$).

From Fig. 2a, it is seen that slot shape 2 has a smaller initial rate of opening than slot shape 2f. Yet, slot shape 2 gives an overexpansion in the centerline Mach number distribution, whereas slot shape 2f gives a smooth centerline Mach number distribution. One may, therefore, conclude that the initial rate of opening is not a major significant factor. It is conjectured that the improvement in the flow quality obtained in many tunnels, thought to be due to a reduction in the initial rate of opening of the slots, is, in fact, attributable to the consequent reduction in the open area ratio in the region where ν_w is approaching ν_p. If one is not limited in the length of the region over which the supersonic flow has to be developed, then it is believed that even a constant width slot shape having an open area ratio of, say, 0.02 would give a Mach number distribution on the centerline (for, say, an $M_p=1.2$) with only small Mach number variations of the order of 0.002. If one desires the flow to be developed in a short region, then an initial quick opening followed by a subsequent gradual reduction in the opening of the slots, similar to slot shape 2f, is needed.

Design Method

In the design method, we prescribe a smooth supersonic flow development in the test section for a design Mach number, and then determine the open area ratio variation needed to generate such a flow. It was found most convenient to prescribe a smooth supersonic flow development in the test section through prescribing a smooth $(\nu+\theta)_w$ distribution along the homogeneous wall ending up with a constant value ν_d, the Prandtl-Meyer angle corresponding to the design Mach number. Typical examples of the $(\nu+\theta)_w/\nu_d$ distribution over the initial developing region are shown in Fig. 8. It may be recalled that $(\nu+\theta)_w$ manifests as ν_{cl}. Hence a smooth distribution of $(\nu+\theta)_w$ implies a smooth variation of ν_{cl} or smooth variation of the Mach number on the centerline. It may be further recalled that once the boundary condition relating ν_w and θ_w is given, the whole characteristic network can be developed and the whole flowfield determined. In particular, the ν_w and θ_w would be determined. In developing the characteristic network, it was again assumed that the slope of the effective homogeneous wall was zero with the expectation that appropriate geometric wall divergence generally can be provided to compensate for the boundary-layer growth. Thus, θ_w, the flow inclination with respect to the centerline, is the same as the flow inclination with respect to the homogeneous wall.

To find the open-area ratio, we make use of the cross-flow pressure drop boundary condition, given by Eq. (5), viz.,

$$\theta_w = KR\sqrt{C_{p_w}}$$

where all the quantities are as defined earlier. To get the ideal open area ratio, K the slot orifice coefficient is taken to be unity and the plenum Mach number is taken equal to the design Mach number. Since ν_w and θ_w have been found from

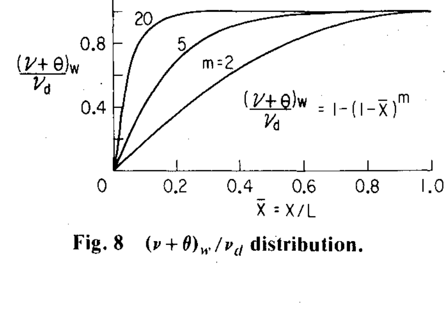

Fig. 8 $(\nu+\theta)_w/\nu_d$ distribution.

the solution of the characteristic network and C_{p_w} is a function of ν_w and the plenum Mach number, all the quantities except R, the open area ratio, are known in Eq. (5). Hence, the ideal open-area ratio variation R_i can be determined. The geometric open area ratio R_k required to take account of the slot orifice coefficient is given by $R_k = R_i/K$. It turns out that the open area ratio R_i and, consequently R_k, end up with zero value at the end of supersonic flow development region as shown in Fig. 9. This is analogous to the wall slope going to zero in a solid supersonic nozzle. However, for a transonic wind tunnel, where the slot shape remains the same for testing at subsonic speeds, the open-area ratio being zero in the test region is not acceptable from the standpoint of model interference considerations. Even for supersonic operation, at off-design Mach numbers, the open-area ratio reaching zero value may not be desirable since it prevents readjustment of the flow, if the flow has not reached uniform conditions by then. Because of these practical considerations, provision has been made to modify the open-area ratio variation R_k to smoothly end up at a constant value R_c desired by the designer, and it is expected that R_c will be less than 0.05. The transition is through a segment of a quadrant of an ellipse with a semimajor axis equal to R_x and a semiminor axis equal to R_R, as shown in Fig. 9. These modifications could result in the performance deviating slightly from the prescribed performance even at the design Mach number. Therefore the performance of these slot shapes at design and off-design Mach numbers was determined using the analysis method to enable making a choice of slot shapes from the consideration of satisfactory performance over the whole supersonic range of interest.

Performance of Slot Shapes Obtained from Design Method

In this paper, slot shapes obtained by the design method for two typical $(\nu+\theta)_w$ distributions are considered. One of these corresponds to the shortest length for the supersonic flow development and the other corresponds to the case where the length for the centerline Mach number distribution to reach the design value is of the order of one test-section height. For both cases the design Mach number is taken to be 1.2. The design and off-design performance of these slot shapes is presented and discussed. No attempt is made in this paper to present an extended study of the various design alternatives to enable choice of slot shapes to be made subjected to the constraints imposed by the designer. However, all the information needed for such a study is provided.

The shortest length for supersonic flow development is obtained when $(\nu+\theta)_w$ is taken equal to ν_d right from the slot origin as shown in Fig. 10a. The ideal open-area ratio and the modified open-area ratio using $K = 0.85$, $R_x = 0.5$, $R_R = 0.05$ and $R_c = 0.025$ for an $M_d = 1.20$ are also shown in Fig. 10a. It might be noticed that for this design the open-area ratio starts off with a miximum value and then gradually decreases. This is analogous to the slope being maximum just after the throat for a solid supersonic nozzle designed for minimum length.

Predicted centerline Mach number distributions obtained with the modified open area ratio, at the design and off-design plenum Mach numbers are also shown in the same figure. A kink in the Mach number distribution at the end of the Prandtl-Meyer flow is observed in all the cases. It is expected that actual flow obtained with a discrete number of slots would probably smooth the kink for the following reason. A rapid variation in the open area ratio, in theory, results in a two-dimensional disturbance being propagated toward the centerline; whereas, in actual flow with a discrete number of slots, the equivalent rapid change in the slot width results in a three-dimensional disturbance and, hence, it can be expected to be more diffused than that predicted by the two-dimensional theory. The Mach number distribution at the design M_p is very good. Although there is a fairly large overshoot and waviness in the initial part of the Mach number distribution for $M_p = 1.10$, it is seen that uniform flow is attained beyond a length of about one test-section height. The Mach number distribution obtained at $M_p = 1.3$ would not be considered satisfactory.

Results obtained with a more gradual $(\nu+\theta)_w$ distribution, again for a design Mach number of 1.2, are shown in Fig. 10b. It may be seen that though theoretically $(\nu+\theta)_w/\nu_d$ reaches the value 1 at a distance of 3 times the test-section height, for all practical purposes one can take $(\nu+\theta)_w/\nu_d$ to have reached the value 1 after 0.8 times the test-section height. Consequently, the design Mach number of 1.2 can be taken to be practically reached at a distance of about 1 test-section height. The open-area ratio obtained is now much smoother. There are no kinks in the centerline Mach number distribution. The overshoot and waviness in the Mach number distribution at $M_p = 1.1$ is now considerably reduced. As discussed earlier, it is expected that the actual distribution that would be obtained in a tunnel with this open-area ratio variation would be even better. For $M_p = 1.30$, it is seen that the centerline Mach number is not able to reach that value, and there is no region of uniform flow. It may be pointed out at this stage that both in design and analysis, the geometric wall divergence is assumed to be such as to compensate for the boundary-layer growth, resulting in zero wall divergence for the homogeneous wall. It also may be pointed out that, from a practical point of view, the wall divergence to compensate for the boundary-layer growth is needed only in the test section region after the uniform flow has been developed and no such requirement is called for in the initial development region. In fact, the rate of growth of the displacement thickness of the boundary layer would be affected by the flow development. In view of this coupling, the authors feel that in an actual tunnel, one can vary the geometric wall divergence to improve the flow beyond that predicted by theory. For example, by providing a slightly greater wall divergence at $M_p = 1.3$ than that found suitable for $M_p = 1.2$, the centerline Mach number

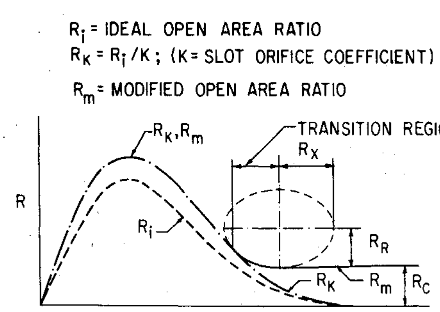

Fig. 9 Modification to ideal open-area ratios.

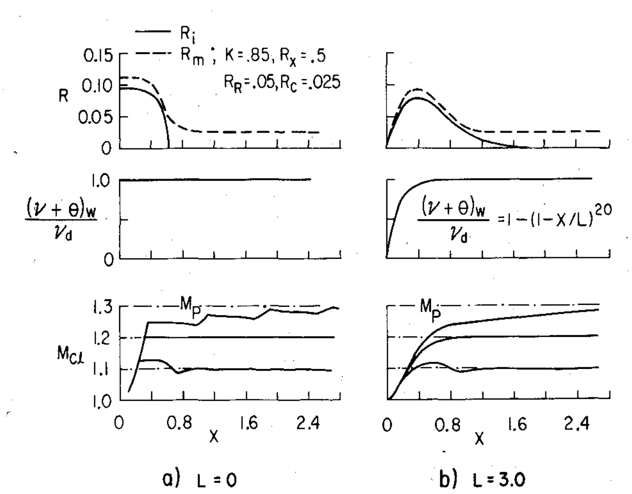

Fig. 10 Designed slot shapes and predicted performance.

distribution may perhaps be made to reach a Mach number of 1.3 and have a region of uniform flow.

It must be pointed out that although no attempt has been made here to cover a variety of design cases, it is believed that the analytical approach to design has been made clear, so that such a detailed design study can be taken up when needed.

Conclusions

A very satisfactory method has been developed for predicting the supersonic flow development in a two-dimensional slotted tunnel when only the slot open-area ratio variation and the plenum Mach number are given. For this purpose, the value of a slot orifice coefficient K in the range of 0.8 to 0.9 is expected to give the best results.

For those cases where the tunnel walls are diverged to compensate roughly for the boundary-layer growth, the assumption of effective wall slope to be zero in the analysis is sufficient to provide satisfactory predictions. However, for those cases where the wall divergence provided does not compensate for the boundary-layer growth, the variation of boundary-layer growth may have to be taken into account explicitly in some manner.

The use of the analysis procedure presented has provided an insight into the basic cause for an initial overexpansion and downstream waviness in the Mach number distribution on the centerline with some slot configurations. To avoid this, it is necessary that the open-area ratio of the slots in the region where the wall Mach number approaches the desired freestream Mach number be kept to a minimum.

An analytical approach to design the slot shapes for a prescribed smooth supersonic flow development at a design Mach number has been developed. Since the design method is essentially an inverse of the analysis method, one can expect the design method also to be satisfactory.

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